THE OPTIMAL ALLOCATION RULE AND THE WATER MARKET AS THE MOST EFFECTIVE TOOLS OF MANAGING WATER SHORTAGE IN AN IRRIGATION DISTRICT

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Abstract

In this work the efficiency of water markets in an irrigation district is put under consideration. This efficiency is referred to the private economic losses arising from a reduction of water availability, so the most efficient or optimal allocation rule will be the one that minimizes those losses or the one that provides the maximum private benefit. Although other rules could be compared with a water market, we focus on this optimal allocation and the proportional reduction as well. The fundamentals of these three practices are included.

Both the proportional and the optimal rule have been applied to an irrigation community and a water market has been simulated, in order to compare to each other in terms of economic efficiency. This has been done solving the respective optimization problem. Results show that water markets will improve the suboptimal allocation made applying proportional reductions, even when transaction costs are high. They also show that the greater the water restrictions, the greater are the gains from trade. It can be inferred too that, as long as all determinants have been taken into account and transaction costs are low enough, the losses of income ensuing from any prescribed water reduction will be the lowest both by means of the optimal allocation as with the market. Anyhow results and conclusions are clearly dependent on the relations made between allocations and yields.

Keywords

Water reduction, water (re)allocation, water market, accumulated loss of income, efficiency, optimization.

1. INTRODUCTION

Regulation of water use may either be direct, through a system of pricing, quotas, or a combination of both; or indirect, with a water market that is more or less institutionally controlled. The possibilities vary depending on the physical characteristics, and the economic, cultural, political, legal and institutional characteristics of each particular case (Tsur, 2009). The widely differing ways of allocating and charging for irrigation water in the world (see, for example, Johansson *et al.*, 2002, or Berbel *et al.*, 2007) thus reflect this variability. The proportional system, the market system and the introduction of a uniform quota rule, developed under the theory of social choice, was compared by Goetz *et al*. (2005), concluding, with a view to economic efficiency, that the water market leads to better overall results, particularly in severe water shortage situations.

Significant evidence on the benefits of market allocation is provided from empirical studies of water markets in the United States of America, Spain or Chile (see f.e. Lee and Jouravlev, 1998). A comparative study of water markets in the states of Arizona, California, Colorado, Nevada, New Mexico, and Utah demonstrated that: (i) these markets appear to be relatively efficient in allocating water among uses recognized as beneficial, with transfer patterns clearly indicating a movement from lower to higher-value uses; (ii) third party effects involving consumptive water users, i.e., return flow externalities, are generally reflected in market decisions and prices, but not so instream flow, water quality and other values that are not represented in water rights; and (iii) water markets typically deviate substantially from the competitive market model, and observed market prices may serve as only a rough approximation of the social value of additional water supplies (Saliba and Bush, 1987; Saliba, 1987; Saliba *et al*., 1987). In southern Spain several studies on hypothetical water markets in irrigation districts have been dealt with (see f.i. Arriaza *et al*., 2002; Calatrava and Garrido, 2005; or Albiac *et al*., 2006). A simulation model comparison of the market approach adopted in Alicante with those found elsewhere in Spain, where trading is not permitted, indicated that the market is the most efficient in terms of net increases in regional income. The differences are not great with moderate water shortages, but significant in conditions of severe shortage (Lee and Jouravlev, 1998).

However, the establishment of a water market demands skills and attitudes from the public administration, judicial systems and water users, as well as investment in registration of rights, monitoring and measurement systems, and possibly in improving water distribution and transportation systems (Lee and Jouravlev, 1998). Efficient construction of any market requires the existence of the necessary conditions for trading to occur: (i) well-defined property rights; (ii) public information on the supply of and the demand for water rights; and, (iii) the physical and legal possibility for trading to take place (Curie, 1985). Assigning well-defined property rights is a basic question, not only with a view to ensuring that the water markets operate smoothly, but also to enable the increasingly frequent periods of drought to be efficiently managed (Lorenzo-Lacruz *et al*., 2013). Any given distribution of endowments (rights) will give rise to one set of market outcomes; but if rights are poorly defined, market processes cannot be relied upon to allocate water resources efficiently. Therefore government policies play a critical role in defining the institutional setting for market operation and provide the basis for market activity by defining, allocating and enforcing water rights (Lee and Jouravlev, 1998).

A continuous trade in water rights generates prices that by coordinating dispersed information and preferences indicate the opportunity cost of water or its relative scarcity.

Price is an information-rich signal which summarizes all information available to market participants and motivates appropriate levels of individual action in response to changing demand and supply conditions, thus performing the crucial rationing function in allocating resources to different uses and users. Thus, transferable water rights create a system of economic incentives in which those who have the best knowledge about returns to water in their intended use -water users themselves- are encouraged to use that knowledge to allocate water to higher-value uses and hence maximize its economic value. The extent to which observed market prices accurately measure the scarcity value of water and encourage its efficient allocation, will depend upon the extent to which the characteristics of the market approximate those of the competitive paradigm (Cummings and Nercissiantz, 1992). The costs involved in the transfer of property, or transaction costs, and the costs of transporting water, can significantly affect this capacity of any market to operate efficiently. Transaction costs prevent the equalization of marginal water values among different uses, users and locations. These price differentials between and among uses, users and locations represent unrealized gains from trade, and hence inefficient allocation.

In constrast to water marketing, *a priori*, any water sharing or allocation could be achieved by imposing suitable quotas or by a combination of quotas and pricing. Allocation rules that take into account the differences in water productivity between the various crop exploitations have been dealt with in Alarcón *et al.* (2014a, 2014b), proving that the most efficient is the rule that promotes crop-specific reductions so as to minimize the sum of all farmers' losses of income within an irrigation community. In fact, irrigation water allocation has been often been modeled with the aim of maximizing the overall economic benefits (Reca *et al*., 2001; Shangguan *et al*., 2002; Benli and Kodal, 2003; Letcher *et al*., 2004; Ortega *et al*., 2004; Babel *et al*., 2005; or Jin *et al*., 2012). Optimal water allocation has also been studied implementing game theory techniques (Sechi *et al*., 2013), fuzzy programming approaches (Lu *et al*., 2009; Wang and Huang, 2012) and fuzzy cooperative games (Sadegh and Kerachian, 2011). Environmental release has even been considered to optimally allocate water in detrimental of irrigated agriculture, combining the drought cost on the environment and irrigation net profit in terms of water use (Grafton *et al*., 2010). There are also plenty of studies looking to maximize the added benefit through modeling a water market. Many of them suggest that market options are more economically efficient than the non-market ones, especially in shortage situations, although the gains from trade may be very different among irrigators (Arriaza *et al*., 2002. Martínez and Gómez -Limón, 2004; Calatrava and Garrido, 2006; Pujol *et al*., 2006; Blanco-Gutiérrez *et al*., 2011).

Although theoretically some goals for water allocation, such as predictability, equity and fairness, and the need to reflect collective, public or social values, might be better served by non-market institutions, the existence of these problems does not necessary call for a non-market alternative (Anderson, 1982). Morover, in cases of water restrictions the sole target of maximizing the aggregated production in an irrigation area may inflict very different losses of income between farmers (Smout and Gorantiwar, 2006; Gorantiwar and Smout, 2007; Alarcón *et al*., 2014a, 2014b). Market transactions are *fair* in the sense that water reallocation takes place through voluntary mutually beneficial trades with perceived advantages for all the parties involved; each party must be made better off or one would refrain from trading. Markets can guarantee fairness only, however, if no single market participant can affect market prices. In addition, unless conducted in an institutional framework which causes market participants to take into account third party impacts, markets generally cannot guarantee fairness to third parties who may be negatively affected by market transactions (Lee and Jouravlev, 1998).

In this paper we focus on a way to obtain the information requirements needed for water to be allocated efficiently. On the whole, information requirements for efficient water marketing are no greater than those needed for an effective administrative allocation of water (Easter, 1994). Besides it is shown herein that intra-sectoral water markets for irrigation can improve substantially the efficiency gained by other allocative mechanisms and approach the most profitable water allocation among the users within an irrigation district. This aproximation will be bigger the lower the transaction costs are, hence a comparisson is made. The optimal water allocation can also be attained through an institutional setting of water rights, as long as there is no informational constraints, making that the marginal benefit gained from every single plot of land within a district is the same. In this paper we study annalytically the way to do this when an irrigation water shortage is concerned.

2. THEORETICAL SETTINGS AND METHODS

2.1. Water benefit functions

The cost to farmers resulting from the implementation of water restrictions can be easily deduced from their net private marginal benefit, *MB*, functions. Many examples of the estimation of the marginal value of water can be found in the literature (Young 2005; Mesa-Jurado *et al*. 2010; or Moriana *et al*. 2003). Constructing yield functions is possible from biophysical simulators of crop growth (e.g. Goetz *et al.* 2005). In practice, where literature does not provide yield functions, they may be estimated by using a specific software package, *CropSyst* or *AquaCrop* FAO crop-model for instance.

In the absence of field data with which one could perform econometric analyses, we propose a less data-demanding method. In a particular irrigation zone, *B* represents the benefit which can be obtained from growing a particular crop. Thus, substracting the benefit that can be obtained without irrigation, B_s, gives us the increasing benefit that is obtained by applying a water allocation, *q*: $IB(q) = B - B_s$. Marginal Benefit, *MB,* is the benefit variation resulting from increasing *q* in a unit.

Although other formulae could be used, for the purpose of simplicity, we will use quadratic benefit functions of the following type:

$$IB(q) = B - B_s = \frac{m}{2} \cdot q^2 + k \cdot q \qquad | \qquad q \geq 0\,,\ m < 0,\ k > 0 \tag{1}$$

$$MB = \frac{dB}{dq} = \frac{dIB}{dq} = m \cdot q + k \tag{2}$$

From this simple setup, it is clear that it will be of interest to irrigate if $MB>0$, although its value decreases as the amount of water applied increases (because $m<0$).

If irrigation water deliveries are reduced, the new allocation for each irrigator, *q*, will be less than or equal to a *reference allocation*, without restrictions, q_r, which must be consistent with the crop requirements to obtain the maximum benefit, B_r. In such a case, the benefit strictly derived from irrigation would be $IB_r = B_r - B_s$. Thus, the maximum point (q_r, IB_r) will be taken as the reference point. Note that the double point condition, mathematical maximum, means that the two coefficients *m* and *k* can be easily determined:

$$MB_r = 0 = m \cdot q_r + k \qquad \rightarrow m = \frac{-2 \cdot IB_r}{q_r^{\ 2}}, \quad k = \frac{2 \cdot IB_r}{q_r} \tag{3}$$
$$IB_r = B_r - B_s = \frac{m}{2} \cdot q_r^{\ 2} + k \cdot q_r$$

So using these reference values, q_r and IB_r, for a given water availability, $q^*=q/q_r$, equations (1) and (2) can be written:

$$IB = IB_r \cdot \left(\frac{2q}{q_r} - \left(\frac{q}{q_r} \right)^2 \right) = IB_r \cdot \left(2q^* - q^{*2} \right) \tag{4}$$

$$MB = \frac{IB_r}{q_r} \cdot 2 \cdot \left(1 - q^* \right) \tag{5}$$

The consequent loss of income derived from any water availability, $q^*<1$, is obtained from:

$$L = B_r - B = IB_r - IB = IB_r \cdot \left(1 - q^* \right)^2 \tag{6}$$

There may be crops with different types of benefit functions within one single irrigation district. Furthermore, there could well be other sectors that are competing with irrigation for their own water requirements, and such sectors might have very different benefit functions. In such cases, the power functions offer considerable versatility and could supplement the quadratic functions well (see Alarcón *et al*., 2014a).

2.2. Water allocation optimization

We may think of an irrigation district where several crops and/or varieties are grown; or even growing the same variety, different yields arise in response to irrigation, as a result of variation in farmers' abilities, soils, radiation, slopes or any other significant technical or environmental factor. Hence, there could be as many benefit functions as resulting from the combination of crop-varieties and farmers.

For any given crop *i*, the percentage reduction of water in a particular year, with respect to a reference year of maximum benefit, has been expressed as $q_i^*=q_i/q_{ri}$. When referring to an irrigation community, we will use bold notes and capital letters. So the percentage reduction of water in a particular year, with respect to a year with no water shortage, will be written as $\mathbf{Q}^* = \mathbf{Q}/\mathbf{Q_r}$.

The rule of proportional water reductions implies that the same percentage reduction of water availability is imposed to the irrigation community as a whole, and so to every crop and hectare: $q_i^* = q_i/q_{ri} = \mathbf{Q}^* = \mathbf{Q}/\mathbf{Q_r}$. Taking this into account, when using quadratic benefit functions, the loss of income of any of the *i* crops will be given by this expression:

$$L_i = IB_{ri} - IB_i = IB_{ri} \cdot \left(1 - q_i^* \right)^2 = IB_{ri} \cdot \left(1 - \mathbf{Q}^* \right)^2 \tag{7}$$

This rule hardly ever bring about the most efficient water allocation. In fact, it will do so only under the unlikely scenario when all growers share the same benefit function and the same initial allocation leads to an equal loss of income per hectare for all of them.

However, the optimal allocation is such that the total income loss, **L**, is minimized or that the aggregate benefit for the community is maximized. The solution is obtained finding q_i^* that makes **L** minimum, with the limitation that the sum of the individual quotas allocated do not exceed the total amount of water available, **Q**.

In the optimal solution, the marginal benefit is the same for all those who receive an allocation. Thus, with quadratic benefit functions the following allocation would be obtained from:

$$MB_i = \frac{IB_{ri}}{q_{ri_i}} \cdot 2 \cdot \left(1 - q_i^*\right) = \mathbf{MB} \quad \rightarrow \quad q_i^* = 1 - \frac{\mathbf{MB}}{2} \cdot \frac{q_{ri}}{IB_{ri}} \tag{8}$$

For the resulting value of **MB**, as all the allocations have to be greater than or equal to zero ($q_i \geq 0 \rightarrow q_i^* \geq 0$), the crops not meeting the condition that $2 \cdot IB_{ri}/q_{ri} > \mathbf{MB}$ will not be considered for receiving a water allocation. Consequently, each value of **MB** will correspond to an availability **Q**, which can be determined adding up the assigned values:

$$\mathbf{Q} = \sum q_{ri} \cdot q_i^* = \sum q_{ri} \cdot \left(1 - \frac{\mathbf{MB}}{2} \cdot \frac{q_{ri}}{IB_{ri}}\right) \cdot \left(\frac{2 \cdot IB_{ri}}{q_{ri}} > \mathbf{MB}\right) \tag{9}$$

Once the value of **MB** has been estimated, expression (9) permits calculating the value of **Q**. Solving the opposite way, that is, knowing **Q**, and then determining **MB** must be done iteratively. Once **MB** is finally known from (9), the crop-specific allocation, q_i^*, can be found with expression (8). The optimum solution is independent of the initial situation: the lowest loss is reached through this optimal rule, regardless of whether or not the initial allocation is the reference situation. If users have other types of benefit functions, other formulation has to be considered so as to make the marginal costs equal for obtaining the optimum yields.

2.3. Water markets

Formulation of a water market

When dealing with intra-sectoral water markets for irrigation, the object function to be maximized is the irrigators' added benefit in an irrigation community, $\mathbf{IB_m}$. The benefit of the farmers who sell water comes not only from the use of their allocations but also from the exchanges made, at a market price, P_m, for every m^3 of water which is sold. If the opposite is the case, the buyers must pay for the new rights that they get, which are bought at the same price:

$$\mathrm{IB_m} = \sum_{i=1}^{n} \int_0^{q_{mi}} MB_i(q)dq + P_m \cdot (q_{pi} - q_{mi}) \tag{10}$$

where q_{pi} is every water allocation resulting from the application of the proportional reduction rule: $q_{pi} = q_i^* \cdot q_{ri}$; refering it to the irrigation community as a whole, $q_{pi} = \mathbf{Q}^* \cdot \mathbf{Q_r}$. q_{mi} stands for each final allocation reached through the market, once the corresponding water rights have been completely traded. Obviously, when $q_{pi} > q_{mi}$, irrigator *i* will be a seller; otherwise he/she would be a buyer, except for the case when no business is made ($q_{pi} = q_{mi}$).

$\mathbf{IB_m}$ will be related to the *MB* crop-specific functions and the water availability after the reduction, $\mathbf{Q}^* \cdot \mathbf{Q_r}$. This quantity should be the same as the one reached through the proportional rule and lower than the reference allocation of the irrigation community. Thus we have the first input constraint: $\sum_{i=1}^{n} q_{mi} = \sum_{i=1}^{n} q_{pi} = \mathrm{Q}^* \cdot \mathrm{Q_r}$. The same way, water allocations must not be bigger than the reference ones with no water shortage: $q_{mi} \leq q_{ri}$. Another constraint in this problem is that every allocation is not negative: $q_{mi} \geq 0$.

Besides, the amounts of water bought and sold ought to be just the same and equal to the total amount of water exchanged in a market where there are *k* buyers (superindex *b*) and *n-k* water sellers (superindex *s*): $\sum_{i=1}^{k}(q^b{}_{mi} - q^b{}_{pi})\cdot A_i = \sum_{i=k+1}^{n}(q^s{}_{pi} - q^s{}_{mi})\cdot A_i$, where A_i stands for the area dedicated to each crop.

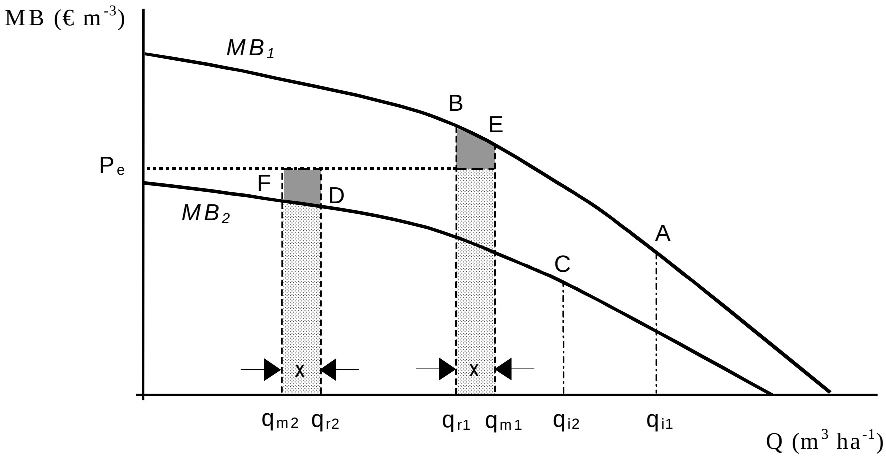
Finally, an *output constraint* has been considered, by means of which each irrigator's benefit reached through the market must be higher than the one attained through the proportional rule: $IB_{mi} \geq IB_{pi}$. In doing so, the optimization problem leads to a solution where an implicit condition is fulfilled for both buyers and sellers: $MB_i=ct$. Therefore the same final solution than the one from the optimal rule should be attained.

Water markets conditions

For a market to ensure flexibility in the allocation of existing water supplies, it is necessary only that within any individual market there is a tradeable margin subject to low-cost reallocation (Howe *et al.*, 1986). This necessary condition is shown in the following Figure 1. It features the marginal benefit functions, *MB*, of two crops, two plots of land, two farms or two irrigators. After applying the optimal allocation, an allocation q_{r1} is assigned to the first irrigator's crop, while a lower q_{r2} is assigned to the second irrigator's crop. So, being crop 1 more productive than crop 2, the irrigator 1 can be interested in acquiring water rights from irrigator 2; and this one in selling them to the first.

In Figure 1 we see that for a certain volume of water exchangeable, *x*, the irrigator 1 has more to gain than lose the irrigator 2 because, comparing the dotted areas in this figure, $q_{r1}BEq_{m1} > q_{r2}DFq_{m2}$. Therefore, there is room for negotiation including an amount equal to the difference between the two areas. The irrigator 2 will sell part of its endowment, $x = q_{r2} - q_{m2}$, for an amount of money that could improve the situation of not selling, i.e., he/she will sell it to a high enough price to offset the loss of income that involves watering less and still make a profit. Similarly, the irrigator 1 will buy rights to a cheap enough price so as the benefit to be achieved with an extra volume, $x = q_{m1} - q_{r1}$, allowes him to make money. The equilibrium point will be in the price, P_e, for which both irrigators earn exactly the same. This extra gain or benefit is represented by the two darkest areas in Figure 1.

Figure 1. Theoretical conditions for the exchange of water rights (*x*), for a buyer (*1*) and a seller (*2*), after a water restriction has been imposed $i \rightarrow r$

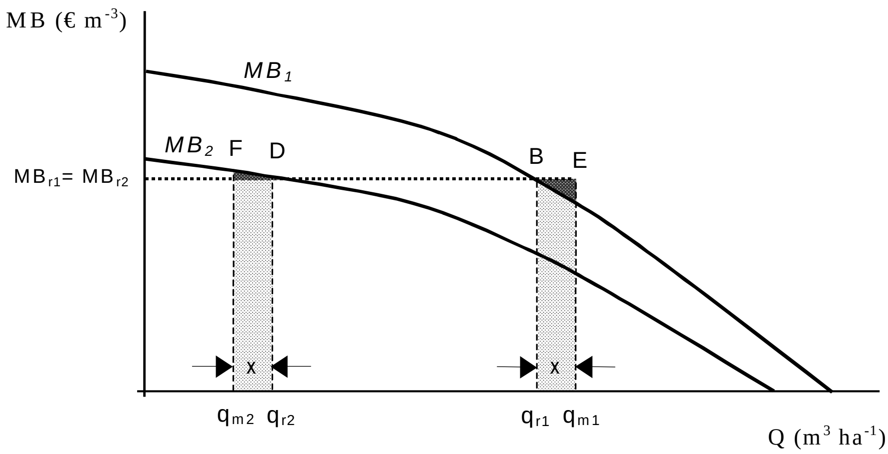


Unless transactions were made directly among irrigators, an organization that would enable them to contact and operate is demanded, a *market* ultimately. Some resources are needed to establish, operate, and enforce a market system which should be paid by the stakeholders. This imply the so called *transaction costs, C_t*. To make it simpler, we may include in them the transportation and infrastructure costs. Transaction costs may be in whole or in part transferred back to trading parties through fees and taxes levied on water rights transfers. In such a way, the price of sale or final market price, would not be P_e, but greater, to include them: $P_m = P_e + C_t$. In spite of this, some components of C_t can be independent of the quantity transferred: the title search, filing fees, and other similar costs often do not vary significantly with the quantity of water being transferred. In terms of equity, transaction costs should be shared equally between the buyer and the seller, in such a way that each should pay the organization a $C_t/2$ amount for the transaction made.

Depending on the negotiation margin and the transaction costs, exchange will prove attractive or unattractive to the farmers. Ultimately, the market will not take place should the marginal values for water, net of transaction and conveyance costs, be equated across water users, uses, and locations. The equalization occurs because the market provides both an incentive and a means for water users to reallocate water rights to higher-value uses whenever reallocation would generate positive net benefits. Thus, if we start with a sub-optimal allocation of water, the introduction of a market would achieve an efficiency nearly optimal. But if, on the contrary, the starting point is the optimal allocation, for which the marginal benefits of each and every one of the irrigators are equal, there will be no room for negotiation, and no market, regardless of the transaction costs that apply, will be able to improve this efficiency.

The latter can be seen in Figure 2 below. Starting from a water sharing for which MB_{r1} = MB_{r2}, neither the irrigator 1 nor the irrigator 2 are interested in buying or selling water, because in doing so, they would incur losses. Thus, if irrigator 1 decides to acquire rights, he/she would have to do it at a lower price than MB_{r1}; but at this price, the irrigator 2 would not be interested in selling. Conversely, if the irrigator 2 sells at a higher price than MB_{r2}, the irrigator 1 would not buy, because he/she would pay more than what he/she gets for having more water. In the extreme case where the market price is equal to the marginal benefit, for an amount of water exchanged, *x*, these two irrigators would face the losses that appear in Figure 2 with a darker shading.

Figure 2. Theoretical conditions of market saturation, for which the exchange of water rights (*x*) does not benefit the buyer (*1*) neither the seller (*2*)



3. CASE STUDY

The previous theoretical analysis has been tested in the irrigation community No. V of *Riegos de Bardenas*, located in the provinces of Zaragoza and Navarra (Northeast of Spain). Because of its large area (15895 ha) and variety of crops, which are allocated different volumes, it provides an insightful illustration of our analysis. Moreover, this district offers sufficient data to implement the analytical procedure. In the annual reports of this community (CRVRB 2011, 2012 and 2013), crop patterns and water consumption referred to recent years are reported. The community report indicated that, in response to lower wholesale district availability, a reduction of 7.29% in the stated average quota was expected from 2011 to 2012.

A baseline scenario was defined by the crop pattern in year 2011, as well as by the allocations under no water restrictions (table 1). These reference allocations were chosen as the largest ones in the series 2006-2012. With them, maximum benefit is supposed to be achieved. Accordingly, it can be said that the water consumption in 2011 was 5.23% lower than it should have been in the reference situation to meet all water demands. For every crop, data to estimate net margin were taken from official statistics (Gobierno de Aragón, 2012) and some technical reports.

Table 1. Baseline scenario in Riegos de Bardenas no. V community

Crop pattern in 2011			Reference allocation (q_r)	Reference irrigation benefit (IB_r)	Reference rainfed benefit (B_s)
Land uses	Area (A) ha	Area (A) %	$m^3\ ha^{-1}\ yr^{-1}$	$€_{2011}\ ha^{-1}$	$€_{2011}\ ha^{-1}$
Alfalfa	2 991	18.82	12 271	823.94	83.35
Corn	3 741	23.54	9 477	213.95	26.42
Cereals	4 905	30.86	3 884	112.55	68.80
Grassland	1 725	10.85	10 689	696.71	430.77
Rice	689	4.33	9 312	776.09	-269.47
Vegetables	487	3.06	10 620	2 222.38	-123.72
Sunflower	221	1.39	6 552	34.16	2.35
Leguminous	342	2.15	2 524	660.37	385.17
Wooded land	171	1.08	2 935	0.00	0.00
Vineyard	29	0.18	736	527.86	1 027.81
Set aside	594	3.74	0	0.00	0.00
Totals	*15 895*	*100.00*	*$124.07 \cdot 10^6$*	*6.884.409*	*1 344 815*

Source: Alarcón *et al.* (2014b).

Quadratic benefit functions were obtained using the method described in section 2.1. Then the proportional rule and the optimal allocation considered in the theoretical sections were applied to the baseline scenario. Afterwards, taking as starting point the water sharing obtained from the former, an internal water market was simulated. In doing so, areas were kept unchanged, included the land set aside. Woody crops were regarded as a social use, and for this reason an exception to the rules: allocations were not changed for them, even if no monetary gains accrued from this policy. The expected 7.29% reduction in the average quota of year 2011 was set for the three allocation mechanisms, except for woody crops. In terms of reference allocations, this means a reduction up to 12% ($\mathbf{Q}^*$ = 88%). Besides, with the aim of testing the effects of the severity in water shortage, a much higher restriction has been dealt with: 25% in the water availability of 2011 ($\mathbf{Q}^*$ = 0.71).

4. RESULTS

The solutions of the corresponding optimization problems for the water restrictions of 7.29% ($\mathbf{Q}^*$ = 0.88) and 25% ($\mathbf{Q}^*$ = 0.71) in the water availability of 2011 were obtained using the Solver tool in the MsExcel package. Otherwise they could also be obtained with a modeling system for mathematical programming problems such as General Algebraic Modeling System (GAMS). The analytical models in Section 2 provide the same results, which have been performed in an Excel spreadsheet using the iterative calculation option and assuming initial values which are automatically corrected by simple procedures.

In the case of the market, the benefit maximization problem has been solved by trying several prices for water exchanged. According to what it has been said in the theoretical sections, after applying the proportional reduction, the greater benefits are achieved at a water price, P_m, corresponding to the average value between the highest marginal benefit, *MB*, of the crop group for which rights are transferred, and the lowest *MB* of the crops for which rights are acquired: P_m = € 0.0115 m^{-3} ($\mathbf{Q}^*$ = 0.88); € 0.0273 m^{-3} ($\mathbf{Q}^*$ = 71). These prices are slightly lower than the *MB* of the entire community, averaged over all crops' consumption, and slightly higher than the median. This is depicted in Figure 3 below, intended for the reduction of 7.29% in the water availability of 2011 ($\mathbf{Q}^*$ = 0.88**)**.

Figure 3. Differentiation between crops for which water rights could be either perceived or transferred according to the distribution function of marginal benefit (*MB*), after a reduction of 7.29% in the water of 2011 ($\mathbf{Q}^*$=0.88) in Riegos de Bardenas no. V community

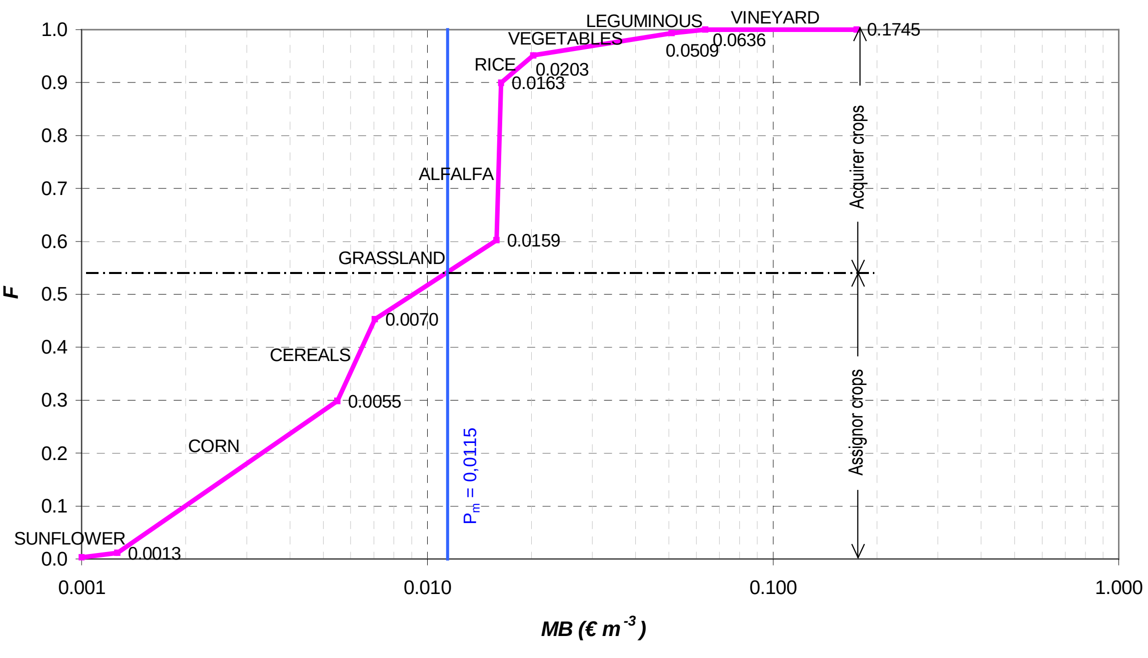
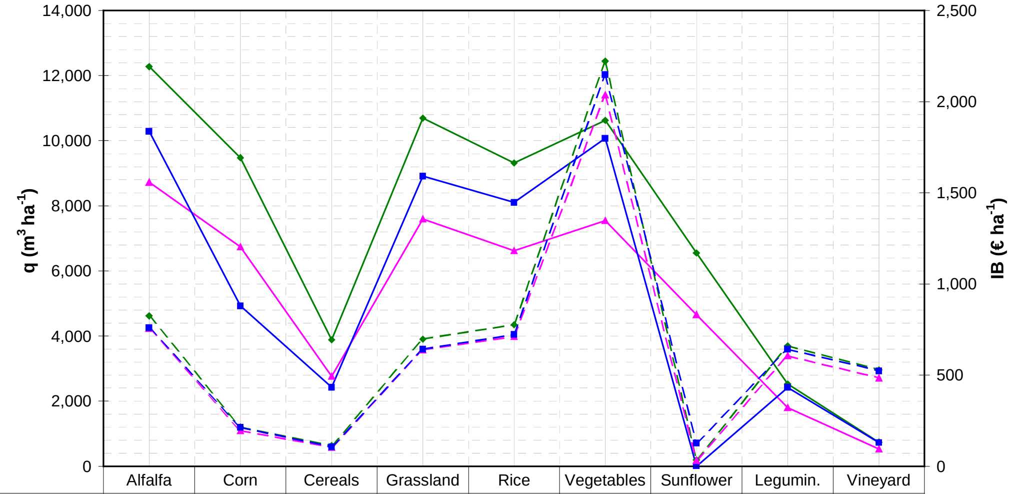


Figure 4 shows water allocations both when no water restriction is considered, that is, the reference allocations ($\mathbf{Q}^*$ = 1), and under the severe restriction corresponding to $\mathbf{Q}^*$ = 0.71. For the latter, allocations have been worked out from applying either the corresponding proportional reduction or a market with no transaction costs, equivalent to the optimal reallocation. The respective irrigation benefits per hectare, **IB**, are also represented.

Figure 4. Water allocations, *q*, and irrigation benefits, IB, under no restrictions and by applying the proportional rule and a market with no transaction costs in Riegos de Bardenas no.V, targeting the same reduction of 25% in the water of 2011 (Q^*=0.71)



	Alfalfa	Corn	Cereals	Grassland	Rice	Vegetables	Sunflower	Legumin.	Vineyard
Reference allocation (q)	12,271	9,477	3,884	10,689	9,312	10,620	6,552	2,524	736
q for Proportional reduction	8,720	6,734	2,760	7,595	6,617	7,546	4,656	1,794	523
q for Market Ct=0 - Optimal realloc.	10,288	4,922	2,430	8,909	8,100	10,069	0	2,419	724
Reference irrigation benefit (IB)	824	214	113	697	776	2,222	34	660	528
IB for Proportional reduction	755	196	103	638	711	2036	31	605	484
IB for Market Ct=0 - Optim. realloc.	760	214	106	642	723	2148	127	642	522

The following table 2 shows the average allocations as well as the average and accumulated benefits for the whole irrigation community, under no shortage (reference situation) and when both reductions of 7.29% and 25% in the water of year 2011 is introduced, either by applying the proportional reduction or a market with zero transaction costs. This best market solution where C_t = 0, coincides with the optimal allocation. A more significant difference in the irrigation benefit, **IB**, arising from one or the other rules is obtained under the most severe shortage.

Table 2. Average allocations and average and accumulated benefits in Riegos de Bardenas no.V, under no water restrictions and reductions of 7.29% and 25% in the water of 2011 (Q^*=0.88 and Q^*=0.71), either by applying proportional reduction or a market with no transaction costs

Scenarios	Average allocations (m^3 ha^{-1})	Average irrigation benefit (€ ha^{-1})	Accumulated irrigation benefit (€)
No water restrictions $\mathbf{Q}^*$ = 1	7 738	455	6 884 409
Reduction for $\mathbf{Q}^*$ = 0.88:			
Proportional reduction	6 860	448	6 782 554
Market C_t = 0 - Optimal reallocation	6 860	451	6 818 743
Reduction for $\mathbf{Q}^*$ = 0.71:			
Proportional reduction	5 803	417	6 307 756
Market C_t = 0 - Optimal reallocation	5 803	430	6 504 525

For this hypothetical market in which there are no transaction costs, the loss of income with respect to the year 2011 as well as the economic efficiency relative to the proportional reduction and the optimal allocation are shown in the third column of the next table 3. In a market like this, perfectly competitive, it would apply the so-called equilibrium price, with zero transaction costs (P_m = P_e). In the following columns of table 3, the values resulting under several transaction costs, C_t, are expressed as percentages of the equilibrium price. As it can be seen, even with very high C_t values, the market would still be more efficient than the proportional rule. The gains from trade under every C_t considered have been calculated substracting the consequent irrigation benefit from reduced water availability to the irrigation benefit reached through a market with no tansaction costs (36187 €; 196769 €). These gains are shown in the last row of table 3.

Table 3. Gains from trade and market efficiency under different transaction costs, C_t, with respect to the proportional reduction and the optimal reallocation, in Riegos de Bardenas no.V, targeting reductions of 7.29% and 25% in the water of 2011 (Q^*=0.88 and Q^*=0.71)

Market attributes	Transaction costs						
	%P_e	0	5	10	15	20	25
Reduction for Q^* *= 0.88*	C_t (€ m^{-3})	0	0.0006	0.0011	0.0017	0.0023	0.0029
	P_m (€ m^{-3})	0.0115	0.0120	0.0126	0.0132	0,0137	0,0143
Water exchanged	hm^3	4.21	4.21	3.86	3.57	3.10	1.27
Loss of income relative to the benefits of 2011	%	0.95	0.96	0.96	0.97	0.98	1.15
Efficiency relative to the proportional reduction	%	100.53	100.52	100.52	100.52	100.50	100.33
Efficiency relative to the optimal reallocation	%	100.00	100.00	99.99	99.99	99.97	99.80
Gains from trade	€	36 187	35 978	35 565	35 278	34 182	22 567
Reduction for Q^* *= 0.71*	C_t (€ m^{-3})	0	0.0014	0.0027	0.0041	0.0055	0.0068
	P_m (€ m^{-3})	0.0273	0.0286	0.0300	0.0313	0.0327	0.0341
Water exchanged	hm^3	9.43	9.43	9.37	8.44	7.36	6.24
Loss of income relative to the benefits of 2011	%	5.52	5.52	5.52	5.54	5.63	5.78
Efficiency relative to the proportional reduction	%	103.12	103.12	103.12	103.09	103.00	102.84
Efficiency relative to the optimal reallocation	%	100.00	100.00	100.00	99.97	99.81	99.73
Gains from trade	€	196 769	196 769	196 745	194 955	189 040	178 930

As it can be observed, the results are sensitive not only to the transaction costs, but also to the reduction in water. Market efficiency with respect to the proportional rule increases accordingly to the severity of the shortage, and much more the gains from trade. In the worst case, where C_t are equivalent to 25% of P_e, the market would still be more efficient than the proportional rule and would reach almost the efficiency of the optimal allocation.

According to Colby *et al.* (1989), in the states of Colorado, New Mexico and Utah, transaction costs incurred by applicants to satisfy state regulation averaged 6% of the prices paid for water rights. In Chile, transaction costs are particularly low in the areas with modern infrastructure and well-developed water users associations: total transaction and transportation were fixed around 2 and 5% of market price; and transaction costs meant around 10% of gross gains from intra-agricultural trades (Hearne and Easter, 1995). As it can be seen in table 4, in our study case transaction costs of 5% of the market price, P_m (in second column), would approximately lead to a loss of only 0.6% of the gross gains, G_m, when Q^* = 0.88, and 0% when Q^* = 0.71. For Q^* = 0.88, 10% of G_m would be lost in response to transaction costs of around 17% of P_m; while for Q^* = 0.71, C_t higher than 20% of P_m will be needed. Therefore, in a rough comparisson with those water markets in Chile, this one in Riegos de Bardenas no.V community could be seen as quite competitive.

Table 4. Transaction costs, C_t, totals and per m^3 of the water exchanged, expressed as percentages of the equilibrium price, P_e, the market price, P_m, and the gains from trade, G_m, in Riegos de Bardenas no.V with a 7.29% and 25% reduction in the water of 2011 (Q^* = 0.88 and 0.71)

C_t (%P_e)	C_t (%P_m)	C_t (€)		C_t (€ m^{-3})		C_t (%G_m)	
		Q^* = 0.88	Q^* = 0.71	Q^* = 0.88	Q^* = 0.71	Q^* = 0.88	Q^* = 0.71
0	0 .00	0	0	0 .00000	0 .00000	0 .00	0 .00
5	4 .76	209	0	0 .00005	0 .00000	0 .58	0 .00
10	9 .09	622	23	0 .00016	0 .00001	1 .72	0 .01
15	13 .04	910	1.814	0 .00025	0 .00051	2 .51	0 .92
20	16 .67	2.006	7.728	0 .00065	0 .00250	5 .54	3 .93
25	20 .00	13.621	17.838	0 .01071	0 .01403	37 .64	9 .07

5. DISCUSSION

As it has been showed, the simulated market achieves the efficiency of the optimal allocation rule, as long as the transaction costs are null. This result was likely expected, owing to that a mathematical problem as the one formulated for the optimal allocation, if well made, has a unique solution and, therefore, this solution cannot economically be improved without changing the problem. Therefore, the implementation of optimal allocation would leave no room for improvement on the market, unless reality provides certain nuances that were not adequately reflected in the formulated problem. Among these aspects not included in the formulation of this optimization problem, perhaps the most influential would be the crop calendars and the seasonality of rainfall.

The results obtained clearly depend on the reference values which are taken for the endowments, q_r, and the benefits from water, IB_r. These values hardly ever will be the same within the same irrigation area, as already noted. When real data provided by the irrigation community are lacked, an approach to the problem of variability in the reference values could be differentiate production rates by crop, as it is made in the Cadastre, but this solution would only involve IB_r values. A sensitivity analysis in determining the allocation of the water applied to the same irrigation district dealt in this paper, under conditions of uncertainty, has been addressed in Alarcón *et al*. (2014b). Specifically, two standard distributions for each crop were devised, with q_r and IB_r as their means, and two respective coefficients of variation: $CV_{qr} = 0.15$ and $CV_{IBr} = 0.30$. With both CV, three q_r and three IB_r values were set, namely $[q_r \cdot (1\text{-}CV_{qr}),\ q_r,\ q_r \cdot (1+CV_{qr})]$ for q_r and $[IB_r \cdot (1\text{-}CV_{IBr}),\ IB_r\ ,\ IB_r \cdot (1+CV_{IBr})]$ for IB_r. Their combination makes nine types for every crop. The area allocated to each crop type was set accordingly to the corresponding probabilities of the bivariate standard normal distribution: 0.204 for $(q_r,\ IB_r)$, 0.075 for $(q_r \cdot (1 \pm CV_{qr}),\ IB_r \cdot (1 \pm CV_{IBr}))$, 0.124 for $(q_r \cdot (1 \pm CV_{qr}),\ IB_r)$ and $(q_r,\ IB_r \cdot (1 \pm CV_{IBr}))$. As it is remarked in Alarcón *et al*. (2014b), with greater CV_{qr} and CV_{IBr}, the best results are obtained, in a sense that the total community losses diminish. As a consequence, searching for more precise data is highly recommended.

6. CONCLUSIONS

It has been shown that the optimal reallocation and the market can correct the inefficiencies of an initial water allocation. In a setting with many buyers and sellers, full information, zero transaction costs, no bargaining and other stringent assumptions, the final allocation of water rights will be the same for the optimal reallocation and the market regardless of their initial distribution. On this view, the initial allocation does not make any difference, but matters to the quantity of transactions or reallocations, the equilibrium allocation of rights, and the aggregate benefits attained with both mechanisms. Since the allocation of water rights may be considered efficient, then close to the optimum, volumes of water that can be expected to be reallocated in response to shortages or changes in the value of its marginal productivity are generally not large.

Both the optimal rule and the market are to be much more efficient than the proportional reduction rule, which in spite of it is being commonly used in collective irrigation organizations. Losses accruing from the application of one mechanism or another may be more or less important, depending on the characteristics of the irrigation community and the severity of the water shortage. Larger efficiency differences between them occur when differences on technical, economic and environmental factors are taken into account. Only in situations with all users being identical, with large water allocations priced at reduced tariffs, will the proportional rule be quite economically efficient. However, under moderate or severe shortages and/or high water rates, other allocation schemes established under the criterion of economic efficiency ought to be encouraged with a view to balance the different interests of farmers.

The optimal allocation rule allows to achieve a specified reduction in water allocation at a minimum overall cost. Its efficiency levels could be such so that there may be no market mechanism that improves it. Besides, the optimal allocation does not involve negotiation costs, and economies of scale could potentially reduce administrative costs in large centralized systems. However perfect knowledge of water allocations and irrigation benefits are demanded, because, as long as there is asymmetric information or uncertainty regarding these values, the optimum would not be attained. Within an irrigation district as large as the one studied, surely there would be important differences in the endowments applied and the benefits obtained of each crop. This expected heterogeneity would imply to consider a greater number of uses and marginal benefits than the ones considered herein. If public authorities had the necessary information to characterize all uses according to reality, they would have the chance to apply the optimal allocation. The difficulties in terms of accurately measuring the crucial variables and functions bring out the point of finding ways to deal with uncertainty, which would likely provide a more fruitful pathway to managing water resources.

Conversely, the market generates some of the necessary information. This an important adventage when such information cannot be acquired at a reasonable cost, when it is fragmented and dispersed among all actual and potential water users, and when demand and supply conditions as well as the value of water are changeable. Another good point in water markets is that the heterogeneity in water uses boosts the participation of market stakeholders and the greater number of transactions will be carried out, thus improving the market efficiency.

The case study has shown that, with low transaction costs, up to 10%, almost the maximum efficiency is reached and that even higher C_t do not detract much. Nevertheless the optimal allocation is preferable when C_t are sufficiently high. It has also been shown that transaction costs of about 5% of the equilibrium price make possible to obtain almost the same efficiencies that null C_t. This fact is interesting, as it provides a useful margin to finance the organization of water markets. By other hand, an increasing water scarcity raises gains from trade relative to the transaction costs. So water markets will become more active where and when water is sufficiently scarce, and hence valuable, since market transactions are precipitated by the difference in the value of water which must be large enough to outweigh the costs of obtaining water through the market process.

Since the future prices in water transfers and the equilibrium allocation are unknown when a decision is made to introduce water rights transferability, the distributional implications cannot be known beforehand. On the whole, there is no particular reason to expect that a water market or even the optimal rule, although providing the greatest economic efficiency, will necessarily result in an equitable allocation of water resources or change income distribution in any particular way. To the extent that water transfers are associated with significant externalities, it is necessary to ensure that market prices will not deviate from the true opportunity cost of water and that the water rights must not come at the expense of society as a whole. Economic efficiency requires that all costs and benefits associated with use and transfer decisions be accounted for. If not, a transfer may be beneficial to the trading parties, but actually inefficient from an overall social perspective.

As long as equity and other important collective, public or social values related to water use are an important part of water policies, it may be necessary to opt for some governmental regulation, in a way that social and enviornmental restrictions can be added to the the optimal allocation rule and the market in order to limit externalities. From the viewpoint of economic efficiency, water rights holders must face the full opportunity costs of their actions, so external effects should be accounted for in transfer decisions. These concerns can usually be accommodated within the logic of the water allocation or market system, for example, by acquiring water rights for a desired purpose, levying taxes on those uses creating negative externalities or paying subsidies to those creating positive externalities. Therefore reliable hydrological information and a knowledge good enough of these externalities are essential to determine who would be affected by a transfer and the magnitude of injury.

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